Dynamic Model of Role-Switching Dictator Games

In order to provide a framework in which to study the pattern of behavior exhibited over the 46 rounds of play in the role-switching game, it is useful to specify a model which incorporates some of the dynamic aspects of the experimental environment. In order to simplify the analysis, certain features of the experiment such as the stochastic assignment of roles are relaxed in the model presented here. The emphasis is instead placed on the relative levels of previous demands made by the Dictator and demands learned during rounds in which the subject was a Recipient.

The situation facing a Dictator in the role-switching experiment can be represented with the following Bellman equation:

$$D(S,P) = \max_{S' \in [0,100]} \left\{ u_D(S', 1_{\{S<P\}}, 1_{\{S\geq P\}}) + \beta \sum_{P'=0}^{100} R(S',P')\mu(P') \right\} \tag{1}$$

The state variables for this problem S and P represent the previous demand (S) made by the current Dictator and the demand (P) observed by the current Dictator in the previous round that the current Dictator was in the role of Recipient. For simplicity, the model assumes that each player switches roles each round, unlike the actual experiment in which roles were assigned randomly subject to the constraints discussed in the paper. Obviously, in an experimental environment of sufficiently short duration β can be assumed to equal 1, but is included to ensure existence of a solution to this dynamic system (which might require $\beta < 1$). With a given previous own demand S and the demand most recently observed while a Recipient P, the Dictator's value function $D(S,P)$ is the value of the right hand side of the function evaluated at its argmax which is the optimal demand S' made by the Dictator today. This choice is a function of the Dictator's current utility generated by the slider choice as well as the continuation value of entering the Recipient role next period having made choice S' with expectations over the slider choice $P^{'}$ made by the next Dictator that the current Dictator randomly meets. The current dictator's beliefs over the next Dictator's slider choice are given by $\mu(P')$.[1] This choice P' will not necessarily be a function of S' due to the fact that anonymity between Dictator and Recipient is preserved so that the Dictator whom the current Dictator meets next period will not have direct knowledge of S'.

The value of being a Recipient in the current period is simply the utility from your partner's choice as Dictator plus the discounted value of being a Dictator tomorrow having made previous choice S and been subjected to the current Dictator's decision P.

$$R(S,P) = u_R(P, 1_{\{S<P\}}, 1_{\{S\geq P\}}) + \beta D(S,P) \tag{2}$$

An additional component of this model that remains unspecified for now is the formation of beliefs over what other Dictators will do. These are especially difficult to formulate when considering results in which

[1] Rabin (1993) and others have allowed beliefs to enter the utility function explicitly.

players are never a Recipient to more than one offer from each Dictator. One idea would be a specification of initial conditions $S_0 = P_0 = S^*$ with S^* being each subject's prior belief of a "fair" distribution of the surplus. For players who begin the game in the role of Dictator, there would be an additional initial condition that $\mu(P') = S^*$ since no information about other players is accumulated during the first round in this case. How beliefs should be formed following this initial specification is a subject of much debate and for now I will remain agnostic on what is a proper specification of the formation of beliefs. However, it seems that given the data generated in the role-switching experiment, beliefs in a rational expectations equilibrium should converge rather quickly to a nearly degenerate distribution at $\mu(100) \approx 1$.

Thus, combining versions of utility functions discussed in the paper with our dynamic model specified in (1) and (2), we have

$$D(S,P) = \max_{S' \in [0,100]} \left\{ \left[2(\sigma 1_{\{S<P\}} + \gamma 1_{\{S \geq P\}}) - 1\right] S' + (1 - \sigma 1_{S<P} - \gamma 1_{S \geq P})100 + \beta \sum_{P'=0}^{100} R(S',P')\mu(P') \right\} \tag{3}$$

$$R(S,P) = \left[1 - 2(\sigma 1_{\{S<P\}} + \gamma 1_{\{S \geq P\}})\right] P + (\sigma 1_{\{S<P\}} + \gamma 1_{\{S \geq P\}})100 + \beta D(S,P) \tag{4}$$

With such a model specified, the goal is then to estimate the magnitudes of the σ and γ coefficients using our experimental data in order to better understand the type of social preferences exhibited in our experiments.